Judgement Aggregation

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Outline – Judgement Aggregation

- Motivating Example
- Formal Framework
- Axioms and Procedures
- Impossibility Theorem
- Summary

Motivation

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- A court of three judges decides on a case of a contract.

  Whether the contract in question has been valid ? (p)
  Whether the contract in question has been breached ? (q)

- The defendant is pronounced guilty if and only if both premises hold $(p \wedge q)$.

Motivation

	p	q	p ∧ q
Judge 1	Yes	Yes	Yes
Judge 2	Yes	No	No
Judge 3	No	Yes	No

Collective Judgement ?

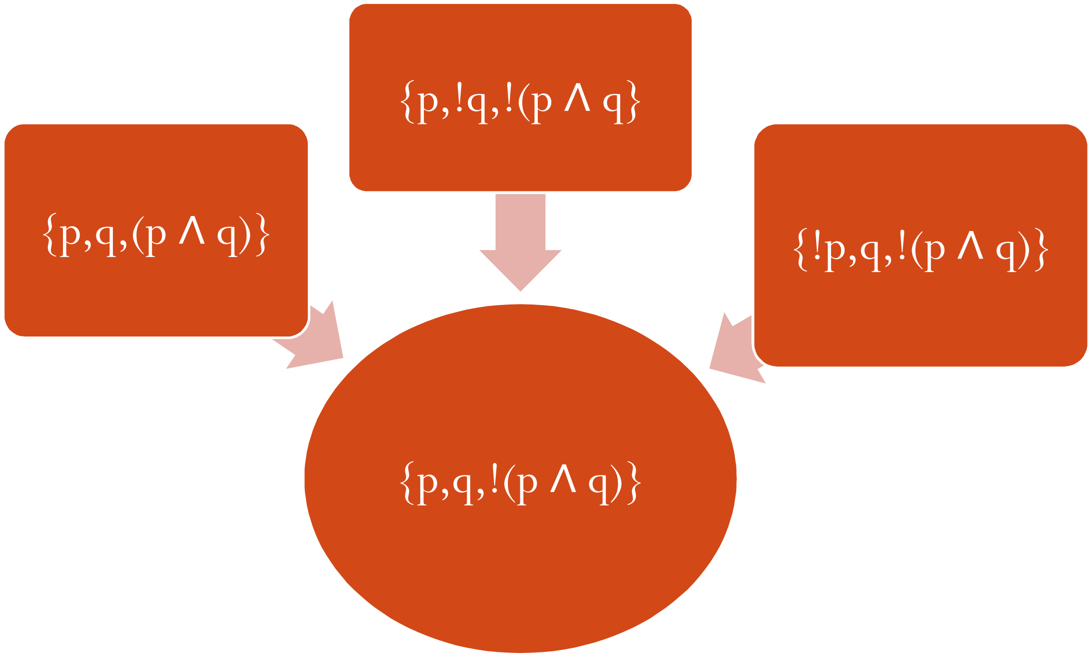
Motivation

	p	q	p ∧ q
Judge 1	Yes	Yes	Yes
Judge 2	Yes	No	No
Judge 3	No	Yes	No

Premise-based vs Conclusion-based

Motivation

?



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Formal Framework

- For any formula φ of propositional logic, let $\sim\varphi$ denote its complement:

  $\psi := \sim\varphi$ if $\varphi = !\psi$, and $\sim\varphi := !\,\varphi$ otherwise.

- Agenda Φ : finite set of propositional formulas closed under complementation.

  $\sim\varphi \in \Phi$ whenever $\varphi \in \Phi$.

Formal Framework

- A judgment set J for agenda Φ is a subset of Φ.
- Set of all consistent and complete judgment sets for agenda Φ is $J(\Phi)$.

J is called complete if $\varphi \in J$ or $\sim \varphi \in J$ for every formula $\varphi \in \Phi$;

J is called consistent if $J \nvDash \perp$

Formal Framework

- Now let N $= \{i_1 \dots i_n\}$ finite set of (at least two) individuals (or judges, or agents)

- Judgment aggregation procedure is a function mapping any profile of complete and consistent judgment sets to a single collective judgment set.

  $$F: J(\Phi)^N -> (2)^\Phi$$

- as Example has shown, if F is the majority rule, then the collective judgment set may fail to be consistent.

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Axioms and Procedures

- Unanimity:
  If all individuals accept a given formula, then so should society:
  $$\boldsymbol{\varphi} \in \mathbf{J}_1 \cap \ldots \cap \mathbf{J}_n \text{ then } \boldsymbol{\varphi} \in \mathbf{F}(\mathbf{J})$$
- Anonymity:
  The aggregation procedure should be symmetric with respect to individuals:
  $$\mathbf{F}(\mathbf{J}_1, \ldots, \mathbf{J}_n) = \mathbf{F}(\mathbf{J}_{\pi 1}, \ldots, \mathbf{J}_{\pi n}) \text{ for any permutation } \pi : \mathbf{N} \to \mathbf{N}$$
- Neutrality:
  If two formulas have the same pattern of individual acceptance in a profile, then both or neither should be accepted:
  $$N_{\boldsymbol{\varphi}}^{J} = N_{\boldsymbol{\psi}}^{J} \text{ then } \boldsymbol{\varphi} \in \mathbf{F}(\mathbf{J}) <-> \boldsymbol{\psi} \in \mathbf{F}(\mathbf{J})$$

Axioms and Procedures

- Independence:
  If a formula has the same pattern of individual acceptance in two dierent profiles, then it should be accepted under both or neither of these two profiles:

$$N^{J}_{\varphi} = N^{J'}_{\psi} \text{ then } \varphi \in \mathbf{F}(\mathbf{J}) <-> \psi \in \mathbf{F}(J')$$

- Monotonicity:
  If an accepted formula receives additional support, then it should still be accepted:

$$\varphi \in J'_{i^*} \setminus J_{i^*} \text{ and } \mathbf{J}_i = J'_i \text{ for all } i \neq i^* \text{ then } \varphi \in F(J) \rightarrow \varphi \in F(J')$$

Axioms and Procedures

- The majority rule, which accepts a formula if and only if a strict majority of the individuals do, satisfies all of the above axioms.

  However, the majority rule may return an inconsistent judgment set.

- The premise-based and the conclusion-based procedures also have a weakness:

  They require to declare which formulas in the agenda are to be treated as premises and which are to be treated as conclusions.

Axioms and Procedures

- Distance-based procedures
- Idea :define a metric on judgment sets that, intuitively, species how distant two different judgment sets are, f.e. Hamming distance H, which is defined as

$$H(J,J') := \frac{1}{2} \cdot |(J \setminus J') \cup (J' \setminus J)|$$

- The distance-based procedure based on H then returns that complete and consistent judgment set that minimizes the sum of the Hamming distances to the individual judgment sets.
- There can be more than one optimal judgment set.

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Impossibility Theorem

- The majority rule will not always return a judgment set that is complete and consistent, in fact

- **Theroem** (List and Pettit, 2002). No judgment aggregation procedure for an agenda Φ with $\{p; q; p \wedge q\} \subseteq \Phi$ that satisfies anonymity, neutrality, and independence will always return a collective judgment set that is complete and consistent.

- Proof : for any anonymous, neutral, and independent aggregation procedure F, collective acceptance of a formula depends only on the number of individuals accepting it. In particular , from the neutrality theorem we have :

$$N^J_{\varphi} = N^J_{\psi} \text{ then } \varphi \in \mathbf{F}(\mathbf{J}) <-> \psi \in \mathbf{F}(\mathbf{J})$$

Impossibility Theorem

- We distinguish two cases:
- (1) Suppose the number of individuals n is even. Consider a profile J under which half of the individuals accept p and the other half accept !p, i.e.,

$$N^J_{\mathbf{p}} = N^J_{!\mathbf{p}} \text{ thus } \mathbf{p} \in \mathbf{F}(\mathrm{J}) <-> \ !\mathbf{p} \in \mathbf{F}(J)$$

- Thus, the collective judgment set must accept either both of p and !p, or neither.
- However the former would violate consistency, while the latter would violate completeness.

Impossibility Theorem

- (2) Suppose n is odd. Consider a profile J under which
  (n-1)/2 individuals accept p and q,
  1 individual accepts p and not q,
  1 individual accepts q and not p,
  and the remaining (n-3)/2 individuals accept neither p nor q.
- Then $$|N^J_p| = |N^J_q| = |N^J_{p \wedge q}|$$
- Hence, either all or none of p, q, and !(p ^ q) must be in F(J).
- If the former is the case, then F(J) is not consistent.
- If the latter is the case, then completeness would require that all of !p, !q, and p^q are in F(J), which would again violate consistency.

Summary

- From contradiction we show for no number of individuals will we be able to devise an F satisfying all three axioms that always return complete and consistent judgment set.

- **The Theorem** is the original impossibility theorem in the field of judgment aggregation.

- The connections between the impossibilities arising in the context of preference aggregation vs judgment aggregation are linked with preference statements such as $x > y$ as judgments that may be true or false.

- While originally associated with problems in legal reasoning and discussed in the philosophical literature, judgment aggregation can have a range of significant applications in other fields, e.g., in the Semantic Web, and more specifically the aggregation of knowledge distributed over a number of different ontologies.

Thank you

for your attention !